



# Stock synthesis: A biological and statistical framework for fish stock assessment and fishery management

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## ABSTRACT

Stock synthesis (SS) is a statistical age-structured population modeling framework that has been applied in a wide variety of fish assessments globally. The framework is highly scalable from data-weak situations where it operates as an age-structured production model, to complex situations where it can flexibly incorporate multiple data sources and account for biological and environmental processes. SS implements compensatory population dynamics through use of a function relating mean recruitment to spawner reproductive output. This function enhances the ability of SS to operate in data-weak situations and enables it to estimate fishery management quantities such as fishing rates that would provide for maximum sustainable yield and to employ these rates in forecasts of potential yield and future stock status. Complex model configurations such as multiple areas and multiple growth morphs are possible, tag-recapture data can be used to aid estimation of movement rates among areas, and most parameters can change over time in response to environmental and ecosystem factors. SS is coded using Auto-Differentiation Model Builder, so inherits its powerful capability to efficiently estimate hundreds of parameters using either maximum likelihood or Bayesian inference. Output processing, principally through a package developed in R, enables rapid model diagnosis. Details of the underlying population dynamics and the statistical framework used within SS are provided.

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## 1. Introduction

Fishery stock assessment models are demographic analyses designed to determine the effects of fishing on fish populations and to evaluate the potential consequences of alternative harvest policies. The conceptual framework for these analyses is provided by simple models of the demographic processes birth, natural and fishing death, growth, maturation, and movement. However, it is rare that sufficient information on all these processes is available over the time period during which a fishery has affected the stock in question. The history of fish stock assessment models was dominated in early years by two, quite different approaches to this problem. One branch of investigation used a time series of an indicator of stock abundance, typically based on the standardized catch rate in the fishery as a proxy for stock abundance, and the time series of fish catch to calibrate a simple two-parameter production model (e.g. Schaefer, 1954) that could provide inference about current and target fish stock abundance and the maximum sustainable yield. The other approach, cohort analysis or virtual population analysis (VPA), depended on a time series of detailed

fishery catch-at-age data to reconstruct the virtual abundance of each annual cohort that had been fished (Pope, 1972; Laurec and Shepherd, 1983). This virtual population analysis approach works best when fishing mortality rates have been higher than natural mortality rates, but only reconstructs the historical abundance and fishing mortality rates. Population productivity is then estimated by fitting a functional relationship between the expected value of annual recruitment and spawning stock abundance.

Over the past 20 years, there has been development of a third approach to the fishery analysis problem. Integrated analysis (IA) (Maunder and Punt, in this volume) takes a more inclusive approach to modeling the population dynamics and utilizing a wide range of available data. A definitive beginning for IA is the landmark paper by Fournier and Archibald (1982). The stock synthesis (SS) implementation of IA began during the early 1980s (Methot, 1986, 1989) following Fournier's pioneering work and adopted many of the characteristics of his approach.

Why “stock synthesis”? The term synthesis is used in the context of development of a new product that is more than an amalgam of its disparate parts. In fish stock assessments, different kinds of data can provide complementary information about the stock of fish, but one source may not be sufficient in itself to provide a complete picture of the stock's abundance and the impact of fishing on the stock. SS inherently blends the population estimation paradigm

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of virtual population analysis with the population productivity paradigm of biomass dynamics models. The population dynamics sub-model of SS is quite comparable to the population dynamics found in most statistical catch-at-age models. However, the observation sub-model of SS is highly developed to use a wide range of available types of data to calibrate the model. The observations that can be included in SS include: fishery CPUE or effort; survey abundance; discards; length-, age- and weight-composition data; and tag-recapture data. Technical details are provided in [Appendix A](#) to update [Methot \(2000\)](#). The estimation of target fishing mortality rates and application of these rates in forecasts of potential fishery yield and stock status have been developed extensively in SS. SS evolved as much from an ecological perspective as from a statistical perspective. Thus, it includes capability to use time series of environmental and ecosystem factors to influence the population dynamics and observation processes over time.

SS has evolved and grown to meet the needs of the fishery stock assessment community over the past near 30 years. 35 stocks in the US, 10 tuna/billfish in three oceans, four European stocks, and 12 Australian stocks have been assessed using this approach by 2012 ([Appendix B](#)). Major milestones in this history are recapped in this paper and the current set of SS features is described. Updates to the technical description will be made available on the web to document the evolving features of SS.

## 2. History

The stock synthesis assessment approach has progressed through three major stages of development since its beginning 30 years ago. These stages include an initial development for northern anchovy, *Engraulis mordax*, in which the basic concept was established, re-development as a generalized model focused on west coast groundfish, and a third stage in which the computer code was translated to ADMB ([Fournier et al., 2011](#)) to take advantage of the power of automatic differentiation.

### 2.1. Anchovy synthesis

The original model for northern anchovy ([Methot, 1986, 1989](#)) melded several diverse data sources. Age-composition data were available from the fishery, but fishing mortality rates were low and aging imprecision was a factor, so virtual population analysis could not be used. An egg and larval survey provided a time series of relative spawning biomass, but surveys were triennial during much of the time period, thus hampering a production model approach. A new Egg Production Method had been developed ([Lo et al., 1992](#)) to provide a fully calibrated, direct measure of spawning biomass, but only a few estimates by this method had been conducted by the early 1980s. The anchovy synthesis model integrated these diverse data types into an age-structured history of the anchovy stock off California.

The anchovy SS model incorporated several factors related to the biology of this fish. A latitudinal gradient in age composition and fluctuating ocean conditions that shifted this gradient north-south exposed a varying component of the stock to the relatively stationary fishery. Time-varying fishery selectivity was incorporated in the model to deal with this phenomenon. Second, biological studies demonstrated a temperature dependence on anchovy maturity ([Parrish et al., 1986](#)). Thus, inclusion of environmental effects (ocean temperature) on biological factors (age-specific maturity) in the model helped to explain the dip and then rapid resurgence of the larval index during the early 1970s. Third, anchovy are a forage fish and a known predator, Pacific mackerel *Scomber japonicus*, was changing rapidly in abundance. Thus, time-varying natural mortality as a function of an external ecosystem driver, the abundance

of Pacific mackerel, was a feature of the model. These concepts of time-varying factors linked to environmental and/or ecosystem factors have remained in SS ever since. Unfortunately, situations in which there has been sufficient information to invoke these features have been rare.

A bit of history is worth noting at this point. The anchovy SS was built at the dawn of the personal computer era. It was written in FORTRAN for hand-built computers termed the “LaJolla Standard” running the CP/M operating system with 8-in. floppy disks as the storage medium. Overnight model runs were normal and the code was laced with line printer statements so that something could be retrieved after various failures. In hindsight, as computers have become orders of magnitude faster, the complexity of our modeling approaches have kept pace so that weekend long Markov Chain Monte Carlo (MCMC) runs now represent the frontier of our patience for waiting for model results.

### 2.2. Moving to groundfish

The second stage of SS evolution began in 1988 as NOAA's Alaska Fisheries Science Center sought improvement to the modeling framework for west coast groundfish, particularly sablefish (*Anoplopoma fimbria*) and Pacific hake (whiting) (*Merluccius productus*).

The principle challenge for sablefish was to find a way to take advantage of the new, but growing, time series of fishery and survey size-composition data. Aging data were scarce and fraught with aging imprecision and bias. The modeling concept developed to deal with this situation treated all composition data as a proxy for the actual, true age-composition of the fishery catch or a survey. A matrix in the model transformed the estimate of true age-composition into an expected value for the composition as measured. The transformation matrix could be a simple 1:1 relationship if the fish aging was asserted to be perfect, could be an age matrix that included aging imprecision and/or bias, or could be a matrix that transformed the age-composition into a length-composition estimate using the growth curve ([Methot, 1990](#)). Also, sablefish were caught with fishing gear that was size-selective for larger sablefish, but older sablefish were known to diffuse into deeper water out of the range of most fisheries, so the sablefish SS model incorporated both size- and age-selectivity that could be applied simultaneously. Finally, use of the size-composition data required a growth curve, but external estimates of the growth curve would have been biased by the inability to account for size-selectivity and aging imprecision. Thus, the sablefish SS model included the ability to estimate growth parameters while accounting for the influence of size-selectivity and aging imprecision. Data that enabled estimation of growth parameters included modes in the size-composition data and observed mean size-at-age data.

The situation for Pacific hake was quite different than for sablefish. Here there were complete age-composition data from the fisheries and surveys extending nearly 20 years and a history of modeling using VPA. The interest was in exploration of alternative modeling approaches, especially approaches that could address the strong inter-annual shifts in stock abundance between the US and Canada in response to el nino – la nina ocean conditions. An SS model was constructed using only age-selectivity in the fishery process. Its innovation was in allowing for multiple stock areas with annually varying proportion of the stock in each area ([Methot and Dorn, 1995](#)).

Since the inception of the sablefish and hake SS models in 1988 and 1989, their use expanded beyond sablefish and hake to most Pacific coast groundfish by the late 1990s. As this usage grew, ADMB was being used by numerous researchers to create other statistical catch-at-age models. The SS models were coded in FORTRAN and used numerical derivatives to obtain the gradient information

needed to iteratively search for the best set of model parameters. The analytically calculated derivatives available through ADMB offered faster and better model performance.

### 2.3. Reborn in ADMB

The SS transition to ADMB provided an opportunity to merge the features of the FORTRAN-coded models for sablefish and hake into a fully generalized framework. SS then inherited the powerful features of ADMB for rapid model convergence and variance estimation through inverse Hessian or Bayesian MCMC approaches. This model, termed SS2, began to be used for west coast groundfish assessments in 2005. Evolution of SS2 was rapid and a major re-work of some internal structures in 2009 led to the awkward renaming as SS3.

A note on SS nomenclature and distribution is worth including here. Today, the model executable code is labeled SS3, but the model framework itself is generically referred to as SS. Upgrade versions are tracked with a specific nomenclature. In 2012, SS is at V3.24f where the 3.2 refers to a major model update involving mandatory changes to input files, the “.x4” refers to the addition of some specific feature usable in particular circumstances, and the “f” keeps track of minor tweaks and fixes. The history of model updates is distributed with SS as an annotated, filterable list, and the specific version information and compile date are output at the top of all SS output files.

## 3. Overview of features

The detailed features and formulas found in SS are described in [Appendix A](#). The sections below provide a brief overview of major features of SS: stock-structure, spawner–recruitment, life history and biology, selectivity, fishing mortality, observation sub-model, fishery management targets and forecasting, variance estimation, and output processing.

### 3.1. Stock structure

The total population can be divided among one to many biological entities, and the numbers-at-age of each entity are tracked over time. Some of these can have unique biology (gender, growth and natural mortality) and some can have a unique season of birth within a year. The total of all entities born within a year is referred to as a year-class or cohort. Each of the biologically or birth season-delineated entities will be referred to as a morph. Each morph can be divided into males and females with gender-specific growth and natural mortality rates. In addition, each morph can be sub-divided into slow-, medium- and fast-growing entities termed platoons ([Goodyear, 1997](#)). The phenomenon of platoons within a morph is not observable. However, the logic of their existence is as biologically simple as the concept that a fish that is near the upper end of the size-at-age distribution one year will be within that upper distribution during the following year. Expected values for all observations are based on summation over all platoons and morphs, except male and female data can be kept disaggregated.

To the extent that size-selective fisheries are implemented, each slow-, medium- and fast-growing platoon will have unique age-specific mortality and survivorship ([Taylor and Methot, in this volume](#)). The age-specific mortality on a platoon is calculated according to the average selectivity-at-age, which is the dot product of selectivity-at-size and the normal distribution of size-at-age for that platoon. Thus, there can be differential survival between morphs with different growth rates and between platoons within a morph.

In a multiple area model configuration, each entity can be distributed across multiple areas at birth and move between areas

according to the unique age-specific movement rates of their morph. The model parameters that control the proportion of the total number of births to each morph and subsequent movement rates are potentially estimable with sufficient data and, like all parameters in SS, can be defined to change over time. If the morph feature is being used with platoons, then the platoons move identically to their parent morph. Size- and age-specific mortality is applied within each area to each entity occurring in that area.

In practice, most SS configurations have just two morphs, one for females and one for males, and only one platoon per morph. More complex situations can be created to more closely match biological reality. For example, a model with two morphs, three areas and four seasons could have one morph recruit in area 1 and the other morph recruit in area 2, then both morphs migrate seasonally into area 3 and then back to their natal area in a later season. Models with five areas have been created for bigeye tuna *Thunnus obesus* ([Aires-da-Silva and Maunder, 2012](#)). Another morph-related concept is hermaphroditism. Just as fish of a given platoon can move between areas at the end of each time step, the hermaphroditic function allows an estimable fraction of the females to transfer into the corresponding male platoon at the end of the time step.

### 3.2. Spawner–recruitment

The spawner–recruitment function defines the linkage between the reproductive potential (total fecundity) of females and the expected total number of age zero animals. In a single gender model, all fish are implicitly female; there is no division into males and females and only total abundance is modeled and reported. Male mature biomass is included in the calculation of reproductive potential in a two-gender model only when hermaphroditism is accounted for. The Beverton–Holt spawner–recruitment function is most commonly used and other available functions include Ricker, hockey stick and a three-parameter survivorship-based function ([Taylor et al., in this volume](#)). In a multi-area model, the stock's reproductive potential is summed over all areas to create a global spawner–recruitment relationship. All female morphs and platoons contribute to the global reproductive potential which will create the next global recruitment of a cohort, which is then distributed among the platoons and morphs. There is no heritability to carry the surviving proportions among platoons and morphs into the next generation.

The output of the spawner–recruitment function is the expected mean number of age-0 animals, not the median number. When lognormal recruitment deviations are applied, they are added to a downward bias adjustment from the logarithm of this expected mean recruitment to account for the expected difference due to the variability among the estimated recruitments ([Appendix A, Eq. \(A.1.7\)](#)) ([Methot and Taylor, 2011](#)). The magnitude of this bias adjustment offset can change during the time series being analyzed. In maximum likelihood estimation, it should approach zero in data-poor historical eras with insufficient data to estimate the time sequence of recruitment deviations. The offset should approach the maximum only where there are sufficient data to estimate the time sequence of recruitment deviations with high precision. Because the population's biomass is dependent on the mean recruitment, not the median recruitment, this offset approach assures that the expected biomass will not change as a consequence of having more data from which to estimate the true recruitment deviations ([Methot and Taylor, 2011](#); [Stewart et al., in this volume](#)). In MCMC estimation, the bias adjustment is set to 1.0 for all years because the estimation procedure will integrate over the full lognormal range of potential recruitments.

The duration of the life stage between spawning and recruitment to the population is not explicitly modeled, so the age 0 animals instantly exist as early as the start of the time step in which

spawning occurs. The cohort begins to experience age-specific natural and fishing mortality and to grow according to the defined growth function at this time. In a multi-season model, the total number of age 0 animals can be distributed among seasons starting with the season in which spawning occurs and ending with the last season before the next spawning event. Thus, some of the age 0 animals have their age 0 event delayed until the start of a subsequent season. These later-born morphs are identified by their birth season. Even if these later-born morphs have the same growth parameters as early-born morphs, their delayed age zero event will cause them to be smaller at calendar age than the earlier-born morphs. However, for the purposes of age determination, all fish born in a calendar year are considered to have the same integer age and to graduate to the next integer age on January 1 of the following year. Care must be taken when assembling data for input to SS to assure that the manner in which integer ages are assigned by otolith readers matches this convention in the modeling of age progression.

### 3.3. Life history/biology

Fish body weight is needed to convert the modeled numbers-at-age into quantities such as total catch biomass or a biomass-based abundance index. SS provides an approach for inputting empirical body weight-at-age observations, but typical applications generate estimated body weight-at-age from biological processes in the model. The empirical body weight-at-age approach is common in statistical catch-at-age models and appropriate when there is high precision in these data. However, if these data are not of high precision, the variance in these empirical body weights-at-age should contribute to variance in model estimates. SS provides the capability to calculate body weight-at-age from a length-at-age growth function for each morph, which itself is based on estimable parameters, a body weight-at-length function, and size-selectivity of the fishery to which that body weight-at-age pertains. This capability was critical for the first SS models for Pacific coast sablefish where age data were uncertain. In this approach, SS calculates a different body weight-at-age for the retained fish versus the discarded fish of each fishery for which a size-based retention function is applied. SS calculates mean body weight at true age for use in population dynamics. Mean body weight (or length)-at-age data can be input to SS for model calibration and will inform the estimation of growth parameters while taking into account effect of size selectivity and aging imprecision. Differences in body weight data between fleets will influence estimation of size selectivity for these fleets. Differences in body weight data between retained and discarded fish will influence estimation of the size-retention function. The full integration of growth estimation, size selectivity and retention estimation, and aging imprecision is a major strength of SS.

Body growth in length is calculated according to a growth function, typically the von Bertalanffy. When the model configuration incorporates time-varying growth, the current year's parameters are applied to the current size-at-age of each biological entity to calculate its growth increment toward the current asymptotic size for that morph. Thus, growth is according to morph and fish cannot shrink even if the asymptotic size becomes reduced below their current size, although in the hermaphroditic model configuration the gender change causes an immediate switch from female size-at-age to male size-at-age so care must be taken in the setup of the growth curves. Another model feature allows for cohort-specific growth rates (but not cohort-specific asymptotic size) which could be relevant if there is density-dependent growth according to relative cohort abundance. Fish of each morph can be distributed across areas in a multi-area model. The growth characteristics are associated with the morph, not with the area in which they reside. However, it is reasonable to create a configuration where each

area has its own predominant morph that then has some degree of movement to other areas. Expected values for data, such as the overall size composition of a fleet's catch in an area, take into account the combined catch across all morphs.

SS follows [Schnute \(1981\)](#) re-parameterization of the von Bertalanffy growth curve to redefine the growth parameters in terms of size at two reference ages. Below the lower reference age, where there presumably is little information about the actual size-at-age trajectory, SS simply applies a linear trend to grow the fish from a small size at the age zero event until they reach the lower reference age of the growth curve. More flexibility in the trajectory of growth can be achieved by using the three parameter Richards growth function, or even more by invoking age-specific growth rates for each of several younger ages.

Age-specific natural mortality, maturity, and fecundity can follow various functional forms or be input empirically ([Appendix A](#)). All of these are defined as model parameters, but only natural mortality can be treated as estimable because there is currently no capability to include maturity-at-size or fecundity data in a manner that would allow internal estimation of functions related to these processes.

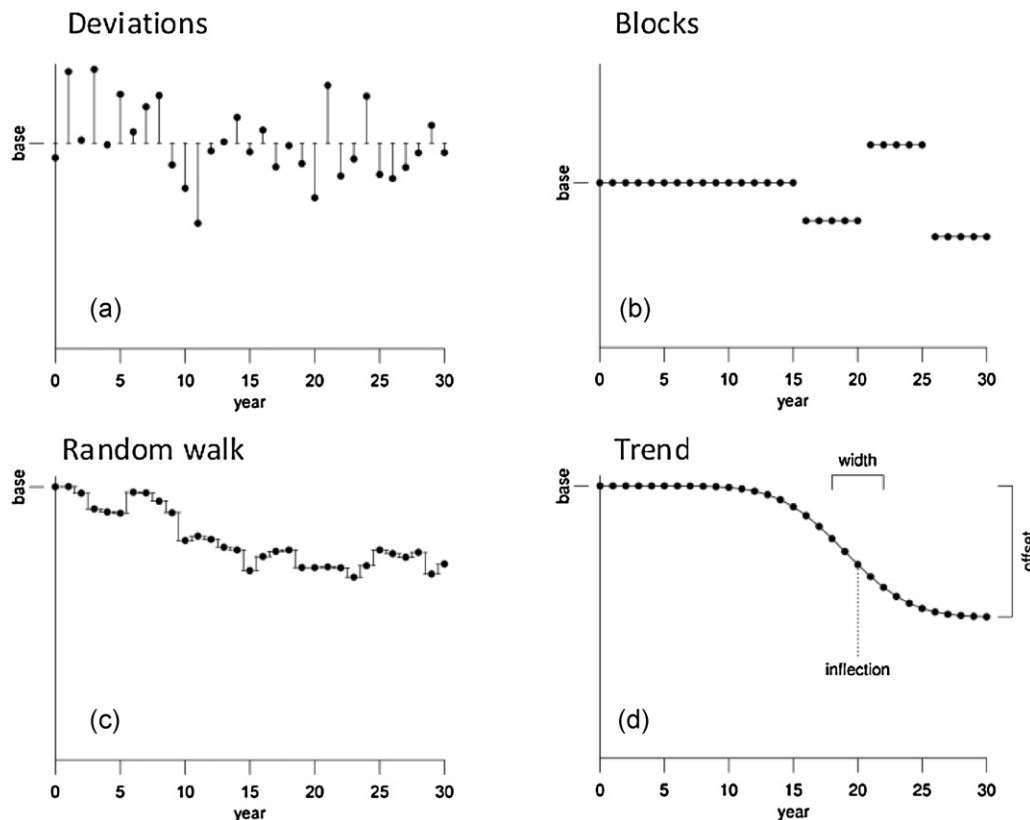
### 3.4. Selectivity

SS provides a range of options for calculating the age-, size- and gender-selectivity of each fishing fleet and survey. SS distinguishes fishing fleets from surveys by specifying that catch is input for fishing fleets, thus the fleet's catch affects the population, whereas a survey is for observation only and does not affect the population dynamics. Most options define a fishery's or survey's selectivity by a smooth parametric function, rather than an age-by-age approach. In common with growth, this approach was implemented in the early model for sablefish where size-composition data were sparse and age-composition data nearly non-existent. The sablefish situation also required the capability for a fleet's selectivity to have age-, size-, and gender-dependency. Size selectivity against small sablefish was logical as they recruited to the longline and pot gear, and declining age selectivity was logical as older sablefish diffused into deep water away from the fishing grounds.

SS includes over a dozen size- and age-based selectivity functions as defined in [Appendix A, Section 5.6](#). These range from simple logistic, to double logistic and double normal, and typically have 2–8 free parameters. The simpler the parametric form for the selectivity, the more the assessment result is constrained by that parametric structure. Similarly, when assessment configurations assert that fishery selectivity is unchanging over time, this is a strong assertion that the fishing fleet is a good sample of the population's size/age composition as filtered by that time-invariant selectivity. [Sampson and Scott \(2012\)](#) provide a sound rationale for doubting time-invariant fishery selectivity. [Taylor and Methot \(in this volume\)](#) show how time-varying selectivity can be invoked in SS to more fully explore a plausible range of model outcomes. For example, selectivity could be near asymptotic during early years when older/larger fish were more abundant, then transition to a more dome-shaped pattern as fishing reduces the abundance of older/larger fish and thus reduces the fraction of the fishing effort deployed in habitats where older/larger fish typically are found. A variety of options are available to allow selectivity (and other) model parameters to change over time. This can be according to user-specified blocks of years, random deviations in selectivity over a range of years, random walk changes over a range of years, a time trend based on a cumulative normal distribution, and as a function of an auxiliary time series, termed an "environmental index". These options are portrayed visually in [Fig. 1](#).

Non-parametric age-selectivity can be used when the maximum age is relatively low and there are reasonably good age data.





**Fig. 1.** Stock synthesis includes five options for allowing parameters to have time-varying values. Shown here are options for annual deviations (a), time blocks (b), annual random walk deviations (c), and Gaussian trend (d). Not shown is the fifth option which links the values for the parameters of the selection function to an environmental time series.

However, the flexibility of size selectivity is not so easily mapped by a natural granularity (large number of length classes implies many additional parameters) to the data. Even more serious an issue would be the very large number of parameters needed for time-varying non-parametric selectivity. The approach of [Butterworth et al. \(2003\)](#) to apply a selectivity surface smoothed in the age and time dimension works well when there are few fleets and reasonably complete data. However, parametric forms are necessary when there are many fleets and sparse data.

A cubic spline selectivity function ([Fournier et al., 1998](#)) achieves a reasonable compromise between non-parametric and parametric selectivity. SS can create starting values for the parameters of spline selectivity functions de novo from the characteristics of the data by aggregating each fleet's or survey's size- (or age-) composition data over all years, distributing the user-specified number of cubic spline knots across that cumulative probability distribution and creating parameter starting values. This capability emphasizes that SS is both a population dynamics model and a software system to implement that model.

### 3.5. Fishing mortality

SS was designed around an approach that relies on the absolute level of catch being known well enough to allow the model to calculate the level of fishing intensity needed to obtain that level of catch conditioned on the model's current estimate of age-specific population abundance and age-specific selectivity. Because the model first calculates catch-at-age in numbers for each fleet, it is straightforward to calculate the total catch for each fleet in terms of both numbers and weight and to provide the capability to use either as the metric in which a fleet's catch is entered into SS. Importantly, this also allows the forecast catch targets for each fleet to be in

either numbers or weight. This is valuable for recreational fishery data which typically is available in numbers caught.

SS does not rely upon fishing mortality rate being estimated as catchability multiplied by fishing effort. Instead, seasonal, fleet-specific fishing intensity is directly estimated to match the observed catch. However, fishing effort and catchability can be brought into the calculations when data on fishery effort or catch per unit effort, CPUE, are provided for parameter estimation purposes. Here, the expected value for CPUE is estimated catchability multiplied by available biomass, and the expected value for fishing effort is  $1/\text{catchability}$  multiplied by fishing intensity.

SS provides three approaches for the calculation of apical  $F$  for each fleet in each time step and area. These are Pope's approximation, which calculates the harvest rate needed to remove the catch at the midpoint of a season, continuous instantaneous  $F$ , with each  $F$  as a model parameter, and a hybrid approach ([Appendix A, Section 1.10](#)) which calculates continuous instantaneous  $F$  values through a differentiable iterative approach. The Pope and hybrid approach treat the harvest rates or  $F$  values as coefficients to be tuned within each model iteration to maintain the match between observed and expected catch, whereas the parameter-based  $F$  approach requires ADMB to estimate these  $F$  values as model parameters (after getting starting values using the hybrid approach in early model phases then converting over to the parameter approach in final stages). The use of the parameter approach to define fishing intensity means that the model will be able to account for uncertainty in the catch data and even allow for missing catch data if there is alternative information such as an effort time series.

A brief comparison of the performance of these three fishing mortality approaches in a simple catch-at-age model with 30 years of catch is informative ([Table 1](#)). The overall negative log likelihood and ending biomass are very similar between methods and

**Table 1**

Comparison among three approaches for calculating the level of fishing intensity in each time step of the model.

	Pope's	Parameters	Hybrid
Number of iterations	458	846	469
–log likelihood	1327.64	1327.68	1327.68
End year biomass	6017.2	5797.9	5798.1
Standard error of end year biomass	1331.5	1294.0	1293.9

the small difference for Pope's method is due to selectivity acting slightly different when used as a mid-year harvest versus continuous harvest (Branch, 2009). The similarity of the standard error for estimated spawning biomass (as determined by the inverse Hessian) is very reassuring, especially given that the parameter approach uses 30 parameters mapped to 30  $F$  values, whereas Pope's and the hybrid method do this using internal coefficients. Although the parameter approach was slower (more iterations) in this example, model convergence can be slow for Pope's and the hybrid method in high  $F$  situations and the ' $F$  as parameter approach' is able to creep up on the final solution more quickly.

The term "fishing intensity" is used because the annual instantaneous rate of fishing mortality,  $F$ , across all fleets is not easily describable by a simple scalar, as it would be in a simple model with only a single fishery and with knife-edged selectivity to this fishery. In SS, there can be many fleets operating simultaneously, some in different geographic zones, and each with selectivity that could vary with age, size and gender. Thus,  $F$  has multiple dimensions. Three options are provided in SS for portraying the overall intensity of fishing: exploitation fraction, equilibrium reproductive output per recruit, and annual numbers weighted  $F$  over a range of ages. Exploitation fraction is simply the total annual catch divided by the total abundance above a specified age, with biomass- and numbers-based options provided. The equilibrium-based quantity is the spawner potential ratio (SPR) (Goodyear, 1993) calculated as the equilibrium level of spawning biomass-per-recruit (SPR) that would occur with the current year's level of fishing intensity relative to the unfished level of spawning biomass-per-recruit. This is reported as 1-SPR to create a metric that increases as fishing intensity increases. All seasonal, geographic, and selectivity issues are integrated into the calculation by focusing on the effect of fishing intensity on spawning biomass. The total  $F$  approaches the problem differently. Here the total annual fishing mortality for each age group (summed over all regions and calculated on a January to January basis) is calculated by:

$$F_{a,t} = \ln(N_{a+1,t+1}) - \ln(N_{a,t}) - M_a$$

where  $N_{a,t}$  is population numbers-at-age, and  $M_a$  is natural mortality rate at age.

SS provides the capability to calculate this  $F$  statistic for a single age, or over a range of ages. The 1-SPR and total  $F$  statistics provide complementary information on the intensity of fishing; one on the cumulative impact of fishing on the entire stock and the other on the fraction of fish removed for key ages.

### 3.6. Expected values for data

Integrated analysis models seek to implement the goodness-of-fit calculations using data that are as lightly processed as possible (Maunder and Punt, in this volume). Some other models, particularly VPA, require a complete catch-at-age and body weight-at-age time series. In some cases these data are created by slicing size-composition data into bins representing the size range believed to represent an age of fish (Kell and Ortiz, 2011). In other cases, missing catch-at-age data are imputed through some interpolation

protocol. Unfortunately, the variance associated with these procedures is not calculated and propagated into the assessment results. The impetus to create catch-at-age data sets has been so pervasive for some stocks in European waters that the details of the historical gap-filling have been lost and it may not be possible to apply a model such as SS to less processed data for these stocks (ICES, 2012).

There is no expectation in SS that data are in the form of catches-at-age, nor that data need to be available for all years of the analysis. Instead, SS takes the data in a lightly processed form and calculates an expected value while taking the various processes that are believed to have influenced the creation of each datum into account. For example, when mean body length-at-age data are input to SS, calculation of the expected value for these data takes into account: the estimated population size-at-age, size-selectivity of the fleet from which the sample was taken, aging imprecision which tends to blur information between adjacent ages of fish, and the estimated recruitment time series.

A powerful feature of SS is its ability to calculate expected values for a wide diversity of data types. Similar capability is found in the CASAL assessment program (Bull et al., 2005). The basic approach is as follows. In each time step, SS tracks the numbers-at-age for each entity by area. SS also tracks the mean and distribution of size-at-age for each entity through time. The vector of numbers-at-age then scales the distribution of size-at-age to create a matrix of numbers-at-size and -at-age for each morph in each area in each time step. Age/size/gender selectivity is then used to create a matrix of selected numbers at size/age for each fishery or survey fleet with data from that area/time. This matrix forms the basis for calculation of expected values for any data type. For each morph, the male and female matrices enable calculation of expected values that are either for combined genders, single genders, or joint genders with preservation of the sex ratio information. For a survey index, the entire selected matrix is simply summed to create the expected value to which catchability is then applied. The size-retention vector is applied to the matrix before calculating the expected values for either the retained or discarded components of the catch if the data are specific to retained versus discarded catch. The aging error matrix is applied before accumulating the expected values into the bins of observed ages if the data are ages. Thus, quantities such as the mean size-at-age of discarded fish can be calculated after accounting for aging imprecision. The various forms in which length, age, weight and abundance data can be represented in SS is outlined in Table 2. A more detailed technical description is found in Appendix A.

An example from the assessment for pink shrimp (*Litopenaeus setiferus*) in the Gulf of Mexico<sup>1</sup> illustrates the approach of fitting data in their native units. Previously, Nichols (1986) had assessed shrimp using a monthly cohort analysis. However, catch-at-month age data are not collected, in fact shrimp cannot be aged using conventional methods. The only composition data were the seven weight categories in which shrimp are sold to the processors. Nichols used a simulation model of shrimp growth and mortality to generate expected proportions at size and then used these proportions to slice the monthly weight composition data into monthly catch numbers-at-age, which were then analyzed using an untuned VPA. SS uses essentially the same process, but in reverse. Within SS, the simulation of recruitment, growth, selectivity and mortality using the parameter estimates leads to an estimate of catch numbers at monthly age. The application of the distribution of size-at-age from the growth curve results in catch numbers-at-length for each month. The generalized size frequency feature of

<sup>1</sup> Hart, Richard. 2012. Unpublished report, Stock Assessment of Pink Shrimp (*Litopenaeus setiferus*) in the Gulf of Mexico.

**Table 2**  
The data types that can be included in SS.

Type	Units	Description
Catch	Biomass or numbers	For each fishing fleet. Initial catch calculation is numbers-at-age for each fleet, then can be aggregated to biomass or numbers. If $F$ is estimated as parameters, then lower precision can be assigned to each catch value. Catch can be size-partitioned into retained and discarded portions, with $F$ tuned to match the retained catch amount
Fleet or survey index	Biomass, numbers, effort	Is aggregated across ages/sizes according to specified or estimated selectivity, so age-specific index is feasible; catchability can be estimated as scaling constant or as a parameter, which can be time-varying or density-dependent; If units are effort, then expected value is $F/q$
Discard	Fraction, biomass or numbers	Created by applying a discard/retention logistic function to the total catch
Mean weight	Body weight	A simple measure of the mean size of fish in the catch (e.g. total weight/total numbers)
Length composition	Proportion	From each/any fleet/survey; combined gender, each gender, or split to preserve sex ratio; from total catch or discard portion or retained portion. Small constant can be added and tails can be compressed
Age composition	Proportion	As with length composition. Also, can be interpreted after applying aging error to expected values. Can be for entire length range, or for specified subset of length range (age-at-length approach)
Generalized size composition	Proportion	As with length composition, but with generalized bins which can be in terms of body length or weight, and accumulated in terms of numbers or biomass in each bin
Mean size-at-age	Body length or body weight	The expected values for these data take into account growth, size-selectivity, aging imprecision
Tag-recapture	Number of tags released and recaptured	Releases are for a specified age in a specified area at a specified time. Recaptures are from a particular fleet (which can only fish in one area) at a particular time
Morph composition	Proportion	In a set-up with multiple growth types (morphs), the proportion of each morph observed (e.g. by otolith micro-constituents) by a fleet/survey can be input
Environmental index	Time series index	Can be input as a “survey” (above) if used as data about recruitment deviations, or can be used as an input driver of a functional parameter relationship

SS then accumulates the catch weight at length into the bins of the catch weight categories to calculate an expected value for the observed catch weight categories. Variance in estimated recruitment, growth and mortality is based upon the goodness of fit to the actual weight composition data. SS is not misled into acting as if monthly catch-at-age data actually exist. The observation sub-model in SS is configured to create expected values for the data, rather than forcing the data to be manipulated to match the structure of the model. This preserves the variance characteristics of the data and allows for estimation of the variance of the processes that created the data.

### 3.7. Fishery management targets and forecasts

SS calculates fishing intensity levels that would satisfy several common fishery management conventions. It does this using a dynamic pool approach (Mace and Sissenwine, 1993) that first calculates the equilibrium level of SPR and yield per recruit (YPR) that would occur if fishing according to a trial level of fishing intensity, then uses this SPR and the unfished level,  $SPR_0$ , to calculate the absolute level of recruitment, spawning biomass, and yield that would occur if fishing intensity were maintained at this rate. SS then iteratively calculates the level of fishing intensity that would match each of three conditions: (1) SPR – a user-specified target level of SPR relative to  $SPR_0$ , (2)  $B_{\text{target}}$  – a user-specified target level of equilibrium spawning biomass relative to the unfished level, and (3) MSY – the level of fishing intensity that would maximize yield. The latter two conditions take the spawner–recruitment relationship into account. These calculations utilize all the multi-fleet, multi-area, multi-season, size- and age-selectivity, complexity in the estimation model, so produces results that are entirely consistent with the assessment result.

SS, in common with many integrated analysis stock assessment models, is basically a simulation of a stock's age-structured population dynamics. This enables SS to utilize a selected fishing mortality approach (e.g. harvest policy) to extend into a forecast of the future age-structured stock abundance and yield that would occur while fishing according to that harvest policy (Maunder et al., 2006). SS

allows the user to specify that forecasting use either the SPR-based fishing rate, the  $B_{\text{target}}$  rate, the MSY rate, or a multiple of the recent average rate when conducting the forecast.

The sequencing within SS calls the fishing mortality target routine and then the forecast routine in two circumstances. First, the routines are not called until convergence occurs and ADMB enters into the variance estimation phase when SS is operating in ADMB's estimation phase. Second, the routines are called during the MCMC evaluation phase so that the equilibrium and forecast results become part of the output for each selected set of parameters. This means that all aspects of parameter uncertainty calculated using the inverse Hessian method in the maximum likelihood estimation, and using the Markov Chain Monte Carlo approach are propagated into the variance of derived quantities, such as the fishing mortality intensity that would produce MSY, and forecasts of stock abundance and future yield that would occur while fishing according to the chosen level of fishing intensity.

Fishery management in the United States has strongly embraced an approach that utilizes catch quotas (termed annual catch limits), and expects that the annual catch limit be set according to a protocol that acknowledges scientific and management uncertainty. A technical approach has been developed to forecast future catch levels that would have a specified probability of allowing catch to exceed the overfishing catch limits (OFL) (Prager and Shertzer, 2010; Shertzer et al., 2010). They calculate an annual acceptable biological catch (ABC) that takes into account scientific uncertainty in the estimate of OFL and an annual catch target (ACT) that further accounts for management uncertainty in controlling catch to the ABC, such that there is a pre-specified probability,  $P^*$ , that attempting to catch that target would in fact result in the fishing mortality rate exceeding the overfishing rate. The flow of an ADMB estimation program does not allow for iteratively searching for target catch levels that would satisfy this probability condition because the uncertainty is not known until the program completes. However, SS takes a multiple pass approach (Appendix, Table A.2) that obtains a nearly comparable result.

The multiple passes in the forecast are needed to mimic the actual sequence of assessment to management action to catch over a multi-year period. The first pass calculates the OFL based on

catching the OFL each year, so presents the absolute maximum upper limit to catches. The second pass forecasts a catch based on a harvest policy, then applies catch caps and allocations, then updates the  $F$ s to match these catches. In the third pass, stochastic recruitment and catch implementation error are implemented and the  $F$  that would be needed to catch the adjusted catch amount previously calculated in the second pass determined. With this approach, SS is able to produce estimates of the probability that  $F$  would exceed the overfishing  $F$ ,  $F_{lim}$ . In effect, it is the complement of the  $P^*$  approach. The  $P^*$  approach calculates the future stream of annual catches that would have a specified annual probability of  $F > F_{lim}$ , while SS calculates the expected time series of probabilities that the  $F$  resulting from a designated harvest policy would exceed a specified level.

### 3.8. Penalized log likelihood

Some data are principally influential in estimating the growth of individuals, others in estimating trends in population abundance, others in the level of fishing mortality, and others in fluctuations in recruitment to the stock. The interactions among these various population and observational factors can be investigated and their combined contribution to the variance in assessment results calculated by including all data in a common framework and using a single overall weighted penalized log-likelihood. Calculation of the log-likelihood for each datum requires an observation error specification to scale all deviations in equal terms.

The error distribution that is typically used for abundance data is the lognormal, although the normal and Student's  $t$ -distributions are also available. The Student's  $t$ -distribution, with a user-specified low number of degrees of freedom, provides a tool to reduce the influence of outliers (Chen et al., 2000).

The multinomial is used for composition data because it inherently assigns more emphasis on fitting data for the composition bins that have higher expected proportions. The multinomial uses the assigned sample size to scale the variance of each bin in a composition vector. This sample size is typically in the range of 50–200 individuals, which is much lower than the actual number of individuals measured because of correlation within the sampling process (Crone and Sampson, 1998). SS allows for a constant to be added to each element of the observed and estimated composition vectors, thus providing increased robustness when data are noisy. SS also allows for the tails of each vector to be accumulated until there is a user-specified proportion in the terminal element of the vector. This tail compression feature causes the model to not make likelihood calculations from a large number of vector elements in the long tail, each of which is near zero, highly auto-correlated, and with low precision. Instead, the accumulated tail will have a higher proportion and thus will get more influence in model fitting. The tail compression feature should be used judiciously when data are sparse because an excessive degree of bin accumulation could occur for composition vectors that are based on just a few fish. A model output based on the multinomial distribution is the “effective sample size” (McAllister and Ianelli, 1997). This value represents the size of the random sample needed, on average, to achieve a fit that is as good as the variance in the model's fit to the composition vector.

The contribution of each datum to the overall log-likelihood function in SS is inherently weighted by the measurement variance of that datum. The user has options to increase or decrease the variances of the input data or, equivalently, the imposed weighting on their contribution to the likelihood function because these variances are rarely known exactly and because extra process error often must be dealt with as measurement error. For abundance data, SS allows for estimation of a parameter for each abundance time series to adjust the standard error of all its observations such

that the root mean squared error of the model fit to that abundance time series is similar to the averaged assigned standard error of each observation. For composition data, input sample sizes should be adjusted to be at approximately the same level as the estimated effective sample sizes to achieve good variance estimates. Tools for iterative adjustment of these sample sizes are provided, but, following Maunder (2011), it may be possible to use the effective sample size calculation in a revised objective function to allow for internal estimation of a parameter representing a sample size adjustment factor.

The objective function in SS contains penalties on parameters and process deviations that are analogous to Bayesian priors, as is common in integrated analysis models (Maunder and Punt, in this volume). These penalties are based on the assumed prior variances for the parameters, and can either be designed to be informative, or be broadly uninformative. Uninformative priors serve to stabilize model performance when data are uninformative about some of the parameters. Examples of informative priors include those for each annual deviation in recruitment being penalized according to the assumed or estimated variability among all recruitment deviations, those for the parameters determining the random walks in a parameter changing over time, and those for parameters such as natural mortality. These penalties are interpreted as priors when the MCMC algorithm is used to sample parameters vectors.

Correct relative weighting among likelihood components is crucial to attain good model performance and good estimates of the variances of the model results. The model result is a complex weighting of the information in the data and the penalties (Methot and Taylor, 2011). For example, when multiple CPUE time series are used as indicators of stock abundance, the final result is a weighted average of all their trends. If a particular fleet's CPUE trend does not match those for the other fleets, its contribution to the overall result is not diminished unless the model is configured to allow for estimation of an extra observation, or process, variance parameter to account for the needed degree of downweighting. It is situation-dependent whether the adjustment should be made on the basis of extra observation variance, which makes the poor fit more likely, or on the basis of extra process variance, which creates a phantom process to bring the ill-fitting data into better conformance with other information in the model. However, the result may not stabilize at an intermediate solution when strongly contradictory information are included in an assessment. Instead, alternative local minima in the likelihood surface may be created and the estimation procedure could be trapped in a local minima and fail to identify the global minima. Patterns in residuals should be scrutinized for evidence that the model configuration has insufficient flexibility to reduce this pattern to white noise consistent with the variance of the data source.

### 3.9. Variance estimation

SS employs several ways to represent uncertainty in the results. ADMB uses the quadratic curvature of the multi-dimensional log-likelihood surface around the maximum likelihood set of parameters to calculate a variance-covariance matrix (Fournier et al., 2011), then uses a Taylor series approximation to propagate this measure of parameter uncertainty to the derived quantities. In SS, the derived quantities include the time series of spawning biomass, recruitment, fishing mortality index, stock depletion, and various reference points such as maximum sustainable yield. The time series of derived quantities extend into the forecast era, providing a continuous sequence of estimated derived quantities and their variances. SS can operate in Monte Carlo Markov Chain mode to provide a Bayesian description of the uncertainty associated with the parameters and derived quantities. It is possible to automate



a sequence of SS runs across a range of fixed values for one or several parameters to generate a likelihood profile or matrix of results.

SS also contains a data generation routine that can provide parametric bootstrap data sets to assist in investigation of model performance and variance estimation (e.g. Piner et al., 2011). The original data are not re-sampled. Instead a new data set is created with the same variance properties that were assumed when analyzing the original data. For each datum, the form and scale of the sampling distribution assigned to that datum (i.e. the distributional assumption of the likelihood function) are used to create a random observation given the expected value for that observation. For example, a multinomial sample of 89 fish would be drawn from the estimated length-composition of a sample that had been given an input sample size of 89. Importantly, the observations in each of these new data sets will not contain any patterns in their residuals, except by chance. Consequently, any autocorrelation or unlikely pattern in the residuals of the original data relative to the model fit will not be propagated into the bootstrap data sets. It is therefore important to ensure that the assigned variance of the input data are approximately the same as the variability between the observed and expected values before creating bootstrap data sets. These bootstrap data sets contain measurement error only and do not randomize any of the process (e.g. annual fluctuations in recruitment or time-varying selectivity) error that may be a feature of the particular model setup being analyzed. Some researchers (e.g. Lee et al., 2011) have used this parametric bootstrap data generation process combined with additional code to allow for process error in processes such as recruitment or selectivity.

It is possible to obtain additional information about the robustness of the model's fit to the original data using this bootstrap approach. When there is pattern in the residuals to the original fit, the model will be in a tension between trying to reduce those residuals while being restrained from doing so by the fit to all the other data in the model and the model structure itself. This tension does not exist among randomly generated observations and the parameter estimates will then tend to drift away from the values obtained when fitting the original data and to a new set of values consistent with the independently distributed data. Following this logic, Lee et al. (2011) and Piner et al. (2011) investigated the robustness with which natural mortality rate could be estimated by SS. Methot and Taylor (2011) used this approach to investigate model performance with regard to estimation of recruitment deviations.

Variance results from parametric, MCMC, and bootstrapping will be quite similar when the normal approximation is good and the residuals in the original model fit are sufficiently random (Figs. 2 and 3). Stewart et al. (in this volume) provides an extensive comparison of these approaches to variance estimation.

### 3.10. Output processing

SS, and all models that deal with complex and voluminous data, needed auxiliary tools to visualize model outputs and quickly produce report-ready tables and figures. This done in SS by providing some output in text tables delimited by keywords and some in list format that provides a filterable/sortable database. This approach allows for parsing by different systems. One is the SS graphical interface developed by Alan Seaver and found in the NMFS Assessment Toolbox (NOAA Toolbox, 2011). A second is an Excel spreadsheet distributed with SS. Third is an R-based module, termed r4ss (Taylor et al., 2011). All three work by searching for keywords, then parsing a table or list of values adjacent to that keyword. This approach provides high flexibility with minimal programming. Users are able to quickly examine results for problematic residual patterns then proceed to produce report quality

graphics using these data display tools. While the Toolbox and R modules have successfully co-evolved with SS, future versions of SS could make the interaction between the input, execution and output modules more intuitive and efficient.

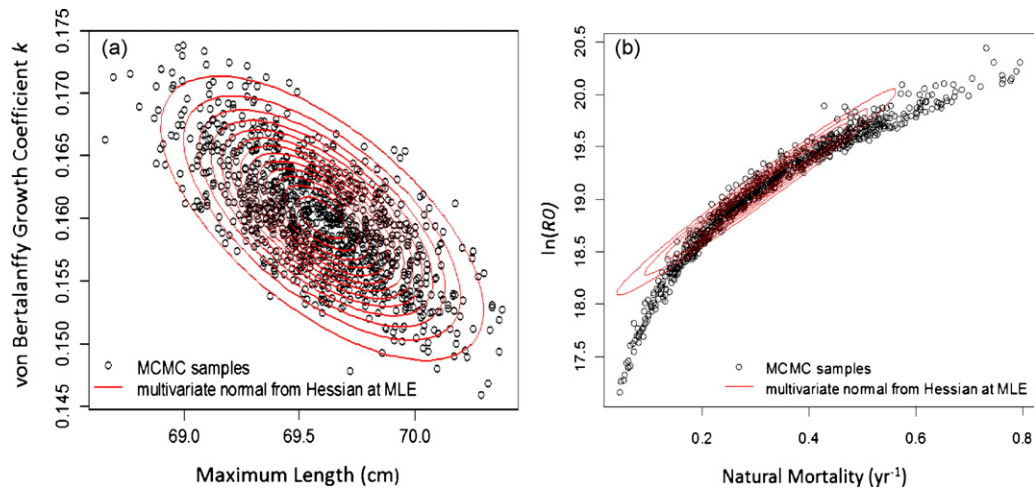
## 4. Discussion

Sixty-one stocks worldwide have been assessed using SS to date (Appendix B), and exploratory applications for many others are underway. This success is due to the flexible scalability to a variety of data and life history situations; to the improved efficiency and communication achieved by use of a consistent, standardization approach; and to the continual evolution to meet the needs of the international fishery assessment community.

### 4.1. Scalable to data availability

The ability of SS to flexibly and simultaneously deal with many types of data is a powerful aspect of the framework. SS is essentially completely scalable between statistical catch-at-age analysis on one extreme, and age-structured, biomass dynamics models and catch curve analysis on the other extreme. SS can estimate the selectivity characteristics of each well-sampled fishing fleet and the time sequence of recruitment to the stock, just like other statistical catch-at-age models when highly informative catch-at-age are available. SS can still estimate selectivity and recruitment, although with less precision, with more reliance on priors and with less signal being detected when composition data are sparse or represented only by length composition. SS can use a fishery selectivity asserted by the user, and collapse the estimated catch-at-age into an expected value for the overall catch and CPUE, thus behaving exactly like an age-structured production model when there is only a bulk indicator of the trend in stock abundance, for example the catch per unit effort (CPUE) of a fishing fleet. If all that exists is a time series of total catch and expert judgment on the degree of stock depletion over time, as in the depletion-based stock reduction analysis (Dick and MacCall, 2011), then SS can use this depletion estimate as a “survey” of spawning biomass at the end of the time series relative to the spawning biomass before fishing started (Cope, 2012). If the only datum was an estimate of the size composition of the catch at one point in time, SS could generate an expected value for that observation while estimating growth parameters and mortality to essentially conduct a catch curve analysis. However, SS could also directly take any information about selectivity and could analyze multiple such observations scattered over the history of the fishery into account, unlike a simple catch curve analysis.

The scalability of SS is facilitated by its definition of annual recruitment as a deviation from the expected recruitment given the spawner–recruitment relationship. All recruitments in a time series can be estimated because the deviations are penalized according to the assumed degree of variability among the recruitments. Thus, SS is essentially a stochastic stock reduction analysis (Walters et al., 2006). Given this approach, data-poor applications end up with near-zero maximum likelihood estimates of all the recruitment deviations, so all the productivity information is just in the two or three parameters of the spawner–recruitment curve. However, the recruitment deviations contribute to the variance to the final result and can take on non-zero posterior values when informative data are included in the assessment. SS partitions the assumed total variability in recruitment into a component that is among the recruitment deviations and a component that is the residual variance of each recruitment deviation (Methot and Taylor, 2011). This feature allows SS to span data-poor and data-rich eras within a single analysis. Thus, SS is scalable to the range of data quality within



**Fig. 2.** Bivariate comparison of the estimated growth parameters maximum length ( $L_\infty$ ) and  $k$  (a). Ellipses represent multivariate normal confidence regions calculated around the maximum likelihood best estimate. The points are drawn from a converged MCMC chain. Two parameters, natural mortality and asymptotic recruitment, for which the normal approximation does not represent the tails well (b).

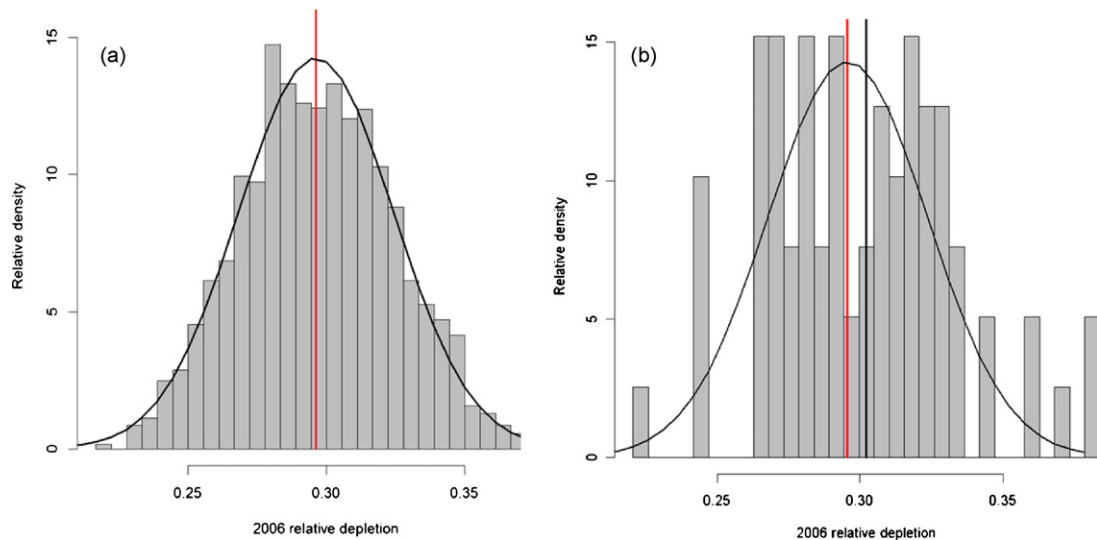
the time series being analyzed, as well as between applications with different data availability.

The value in using a complete model such as SS to analyze data-weak situations is that it is not necessary to switch modeling approaches as more types of data become available. Further, it forces attention on each aspect of the biologically structured population (natural mortality, growth, reproduction, selectivity, etc.) rather than collapsing these processes into a two-parameter black box as in biomass dynamics models. An informative situation was investigated during the 2012 review of assessment models for various flatfishes (ICES, 2012). The assessment had previously been conducted using ASPIC (Prager, 1994), which provides a biomass dynamics approach, with no process error, for analyzing time series of catch and indexes of stock abundance. Alternate assessment models being considered included SS and a biomass dynamics model with process error. SS was used to demonstrate how the biomass dynamics model with unconstrained process error could drift into a high productivity, high process error scenario that fit the abundance index too well. With SS, this phenomenon could be replicated, and then controlled

for by using sufficient length-composition data to stabilize the result to a better compromise between process and measurement error.

#### 4.2. Model complexity

The complexity of integrated analysis models and their ability to simultaneously analyze several diverse types of data means that investigation and control of their performance is challenging. Without informative data, unconstrained model complexity will always lead to highly uncertain results. Simpler models fit to good data can outperform more complex models. However, simple data do not necessarily imply good data nor does a simple model necessarily imply an accurate model of the relevant processes. Good data must accurately represent the state variable (fish abundance) with good fidelity, and the contrast in these data over time must be sufficient to provide information on the dynamics of the system. If data that meet these criteria are available, which is rare, a model that is just complex enough to represent the main dynamic processes should be able to track these data and



**Fig. 3.** The MCMC distribution of relative depletion from the 2006 Pacific whiting assessment (Helser et al., 2006) is a nearly exact match for the parametric estimate of variance in this quantity (a). The right panel (b) shows good comparison between the normal error in this quantity and the results of 75 bootstrap runs. Dark vertical line is the median of the bootstrap estimates.

provide reasonable inference about the population. However, difficulties in measuring fish abundance mean that the time series of abundance indices commonly available for stock assessments are typically contaminated by other processes (dome-shaped selectivity, time-varying selectivity, density-dependent catchability, etc.), and the ecosystem processes affecting fish populations are more complex than a two-parameter parabola. More complex models are needed to take a structured approach to investigate and then account for the many factors that have created patterns in the available data. SS provides this biologically structured approach in the form of size-selectivity, aging imprecision, retention functions, flexible approaches to time-varying parameters, and other factors designed to extract information on fish stocks from a diversity of data types.

SS and all integrated analysis models must be configured to achieve a good balance between data and model complexity. Results are based on the weighted influence of all the data included in the model. The model does not ignore data that do not fit well. It tries to fit each datum in accord with the measurement error associated with that data type, and it uses all free parameters to attempt to improve the goodness of fit. When the data contain patterns that are more complex than can be produced by the model, it will see this complexity as observation deviations from its too simple structure. It will then tend to inflate its estimate of the observation error associated with these data to resolve the lack of fit, and/or it will use other, inappropriate, parameters to attempt to fit the data. For example, consider a model that is presented with complete catch-at-age data that comes from a fishery with a trend in selectivity over time, but the model is not configured to allow for time-varying selectivity. Two things will happen: the time series of estimated recruitments will be biased by trying to mimic time-varying selectivity, and there may be an inflated variance estimate for the age-composition data because recruitment trends cannot fully capture time-varying selectivity. On the other hand, attempting to estimate unconstrained time-varying fishery selectivity in a situation with weak or no age-/size-composition data will result in broad confidence intervals, if model convergence can even be attained in this over-parameterized situation. Model complexity must be appropriate given the available data. One way to accomplish this is to allow the model to have substantial flexibility, mostly as time-varying process error in various parameters, but to also include informative priors on the extent of variation to stabilize estimation when data are not informative. Specification of these informative priors is one of the greatest challenges for good practices in the application of complex fishery assessment models.

Evaluation of the appropriate degree of model complexity is not easy. For example, what are the implications of allowing for dome-shaped selection patterns that change over time, ignoring aging imprecision, or allowing the model to estimate natural mortality rates? Each of these, and many other, questions can be investigated by using one model to generate data with known characteristics from a simulated population, and another model to attempt to estimate that population using the generated data. Ideally, the data generating system and the estimation model would be implemented independently to help assure a robust test of the issues being investigated. He et al. (2011) and Helu et al. (2000) used such an approach to investigate the ability of SS to estimate selectivity. However, consistency between the data generation and estimation models can help ensure that performance is due to the ability of the estimation model to extract information from the data and not due to subtle differences between the generation and the estimation models when the issues are mostly about the ability of the estimation model to derive information from particular types of data or to operate in situations with limited data contrast. The parametric

bootstrap feature in SS provides a tool to generate the needed data sets.

#### 4.3. Linkage to the ecosystem and the environment

An important feature of SS is its ability to serve as a bridge to environmental and ecosystem investigations. The earliest application of SS (Methot, 1986) included the influence of a predator on natural mortality as well as a temperature-effect on maturation, which led to a time-varying linkage between the adult stock and the expected larval abundance survey. Predators can be included in SS because they effectively operate as an additional fishing fleet. If the predator's total annual consumption is known, then this is the catch. If the predator abundance time series is known, then this serves as an index of the predator's "effort". If the ages or sizes of the prey are available from stomach samples, then this serves as the size-/age-composition of the predator's catch. Livingston and Methot (1998) took such an approach in investigating the role of several predators on walleye pollock *Theragra chalcogramma* off Alaska.

The time series of annual fluctuations in recruitment is one of the key outputs from assessment models and the possibility of including environmental covariates in assessment models has attracted much attention in the literature (Maunder and Watters, 2003; Haltuch and Punt, 2011). Where sufficient age- or size-composition data exist, age-structured models such as SS are able to estimate the time series of recruitment empirically. However, estimation of recruitment fluctuations degrade when data quality is poor, as occurs during historical periods and certainly as one extends into the forecast period (Maunder et al., 2006; Methot and Taylor, 2011). In some cases, environmental correlates of recruitment have been surmised and these environmental factors, such as ocean temperature or ocean winds, are often available further back in time than age-structured fishery data, and are available with only short time lag up to present. Environmental correlates of recruitment fluctuations can be included in SS in two ways (Schirripa et al., 2009). In one approach, the environmental time series drives the expected recruitment deviation through an estimated link parameter. Most parameters in SS can have such an environmentally driven link. In the other approach, the environmental information is treated as data relevant to the expected magnitude of the recruitment deviation, rather than a causal factor for the deviation. This approach is more robust when there are missing values in the environmental time series. Brandon et al. (2007) used such an approach in a model of the gray whale *Eschrichtius robustus* population off the west coast of North America. Conceptually and algebraically, this is tantamount to asserting that measuring ocean temperature (or some other relevant environmental factor) is equivalent to conducting a survey of the survival rate of pre-recruit juveniles. In both cases, the information is interpreted to be an index, with measurement error, of the actual recruitment. With small modification, SS could evolve to include this "environment as data" approach for more model processes that exhibit change over time.

#### 4.4. Benefits of standardized models

A comprehensive modeling framework such as SS enhances communication, efficiency, and education in the fishery assessment community (Methot, 2009). Communication is enhanced by creating a familiarity among users, reviewers, and clients regarding terminology and approach. Reviewers who are already familiar with SS can quickly focus on key issues for the assessment being reviewed, rather than spend time learning the features of a novel assessment model. Imagine an assessment workshop with three assessments each with a different modeling system and each with discards treated in a different manner. Communication chaos will

ensue and detract from the effectiveness of the workshop. Standardized models also enhance communication and efficiency by stimulating development of a mature set of tools, such as the output processing package *r4ss*. These standardized approaches and products will reduce the time it takes to produce an assessment report.

Students and stock assessment training can also be beneficiaries of standardized assessment approaches. Numerous students at School of Aquatic and Fishery Sciences, University of Washington, Seattle, WA (e.g. Wetzel and Punt, 2011; Garrison et al., 2011) and elsewhere have honed their programming and population dynamics skills by developing simulation models to test the performance of SS in various situations, as well as to be involved in stock assessments that use SS (Stewart et al., 2011).

The challenge for the fishery assessment, research and education community is to find the right balance between assessment model standardization, which enhances communication and efficiency, and assessment model development, which provides for highly focused investigation and enhances research opportunities and education. Achieving the benefits of both standardization and innovation will require more discipline from the community to use only a core set of models to produce assessment results and to refrain from conducting research as part of assessments designed to inform management. The solution also needs to encourage research into better assessment approaches and more complete testing of model performance to achieve a set of good practices for application of the standard models. The third step would be creation of a pathway by which the results of the research will periodically be reviewed and integrated into the standardized models. A suite of standardized models are commonly employed in climate and oceanographic settings. It seems advisable for the fish assessment community to move in this direction to achieve improved throughput and communication.

#### 4.5. Future evolution of SS

The features of SS have never been static for long and continue to evolve in response to the needs and ideas of the fishery assessment community. All features are contained within one ADMB file with approximately 20,000 lines of code; there has been no splintering into divergent pathways to meet individual needs. While it might seem that this large internal structure containing numerous arrays dimensioned at runtime would result in slow execution, in fact the speed of execution depends principally on the complexity of the user-defined set-up and the amount of the data. A simple age-structured production model will run to completion in a few seconds, while a multi-area model containing rich size-composition data from numerous fleets will take several seconds for each iteration of the estimation procedure and many hours to converge. Undoubtedly, customized models for a particular assessment will execute faster than SS on the same problem, but the overhead to attain full flexibility is small.

A recap of some of the features introduced into SS in 2011 illustrates the pace of evolution.

- a new survival-based spawner–recruitment relationship was used for dogfish *Squalus suckleyi* (Taylor et al., in this volume);
- the degree of depletion of the spawning stock can now be used (Cope, 2012) in the same manner as in Depletion-Corrected Stock Reduction Analysis (Dick and MacCall, 2011);
- a cubic spline selectivity function was introduced to allow more flexibility and potentially produce multi-modal patterns of size-selectivity. This feature was used during the 2011 assessment of west coast sablefish *Anoplopoma fimbria* (Stewart et al., 2011).
- age-specific growth coefficients were introduced to allow evaluation of unexpected patterns in mean size-at-age, as observed

in some tuna species, and comparison to results explainable by size-selectivity alone;

- the capability to create an aging matrix from estimable model parameters was introduced in recognition of the growing interest in accounting for aging imprecision and bias (Cope and Punt, 2007), and was used in the assessment of Pacific cod *Gadus macrocephalus* (Thompson et al., 2011).

Evolution will continue to occur to incorporate new research findings. In some cases, SS itself has been the platform by which the new concept has been investigated. For example, Piner et al. (2011) used the parametric bootstrap feature of SS to provide an approach for investigating robustness of model results. Methot and Taylor (2011) used SS to investigate the relationship between data quality, recruitment estimation, and bias in time series. These findings were made through use of SS, but each is applicable to all integrated analysis type models.

Many ideas are under development. Some of these are:

- alternative likelihood functions for composition data (Maunder, 2011) that would allow for estimation of the variance;
- area-specific spawner–recruitment relationships, rather than the current global relationship;
- explicit elapsed time between spawning and recruitment to more closely match fish life history;
- priors on derived quantities such as selectivity;
- use of environmental time series as indicators of time-varying processes;
- internal estimation of process error for time-varying processes;
- a smoothing algorithm in the forecast so that the target catch one year cannot change too rapidly from the previous year's catch, as commonly is implemented in management procedure (Butterworth and Punt, 1999).

While addition of new features is the most common request from SS users, another is the need for practices to use the flexible features of SS and other integrated analysis models. Too often, a new user starts by invoking more SS features than are supportable by their data and by not taking a structured approach that starts from a simple set-up and gradually adds complexity and flexibility to investigate the phenomena that created their data. User guides to assist in best practices are needed as SS expands to be used by more broadly.

SS, in common with nearly all fishery assessment models, is the product of a single researcher who started out to build an analytical tool for a particular application. While such an approach is common and well-suited to scientific investigations, the importance of the results of stock assessment for government regulatory processes would seem to justify a more concerted effort. As the ADMB-based version of SS reaches its 10 year mark in 2012, it seems time to pause and take stock of alternative development pathways. One is status quo in which gradual evolution of the code is undertaken and additional scientists slowly get engaged in more aspects of code development. Another would be to embark on a more concerted effort to assure that a successor model would even more completely meet the needs of the assessment community. This effort could involve a broadly based steering group to provide guidance for model development, and professional programmers to turn their ideas into well-written computer code. It would take a serious look at the pathway from raw data to model inputs so that re-configuration of model inputs is streamlined and not error-prone. It would co-develop the model engine with the user interface so that control of the model is intuitive and straightforward. The model engine itself would need a strong flexible framework and modular implementation to ease evolution of new features. Assembling the funding and infrastructure to implement and maintain such a



collaborative model will not be easy. However, highly publicized controversies over assessment results highlight the importance of putting sufficient effort into building the best tools possible to take advantage of the great wealth of fishery and survey data being collected today.

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## Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at <http://dx.doi.org/10.1016/j.fishres.2012.10.012>.

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