Hendricks Mark 3 Custom Made Banjo
A Perspective On Steepness and Its Implications for Fishery Management

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A perspective on steepness, reference points, and stock assessment

Marc Mangel, Alec D. MacCall, Jon Brodziak, E.J. Dick, Robyn E. Forrest, Roxanna Pourzand, and Stephen Ralston
Outline
• Density Dependence and Population Biology
• What is Steepness?
• Strategic Fishery Management: The Stock Assessment Process
• Connections Between Steepness and Reference Points: Previous Observations
• The Reproductive Biology of Steepness
• Implications for Reference Points
• Fixing Steepness at 1
• Moving Forward
• Conclusions
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Density Dependence and Population Biology

**Fundamental Law of Population Biology**

Next year’s population = This year’s population
- + Reproduction
- + Immigration
- - Death
- - Emigration
Density Dependence and Population Biology

**Fundamental Law of Population Biology**

Next year’s population = This year’s population
  + Reproduction
  + Immigration
  - Death
  - Emigration

Reproduction ("Recruitment")

= Function of this year’s population size
  and reproductively active individuals ("Spawners")
Because of Density Dependence There Are Points in Biomass-Recruitment Space That Lie Above the Replacement Line
But The Data Are Noisy...

Walleye Pollock (Kamchatka)

Recruits (10^6 Age 6)

10^6 Fish Age 7+

Alaskan Pink Salmon

Fry

Females

Classical Solution: Parametric Stock-Recruitment Relationships -- The Beverton Holt SRR

Recruits, $R$

Spawning stock biomass, $B$
Classical Solution: Parametric Stock-Recruitment Relationships

Spawning stock biomass, $B$

Recruits, $R$
Classical Solution: Parametric Stock-Recruitment Relationships

Recruits, R

Spawning stock biomass, B
The Ricker SRR

Recruits

Spawning stock biomass
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What is Steepness: Mace and Doonan (1988)

Steepness: Fraction of the recruitment at the unfished the spawning biomass when the spawning biomass is 20% of the unfished size

\[ h = \frac{R(0.2B_0)}{R(B_0)} \]

What is Steepness: Mace and Doonan (1988)

Steepness: Fraction of the recruitment at the unfished the spawning biomass when the spawning biomass is 20% of the unfished size

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Parameters of the SRR can be related to steepness and unfished biomass and recruitment

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Strategic Fishery Management: The Stock Assessment Process

• Organize all of the data (catch, survey, fishery dependent, fishery independent)

• Fit the data to a model of population dynamics

• Use the fitted model to investigate state of the stock and management options
Reference Points of Strategic Fishery Management

Unfished Biomass

Biomass giving Maximum Sustainable Yield (MSY)

B_{0}

B_{MSY}

B_{MNP}

Rate of Fishing Mortality giving MSY

F_{MSY}

Spawning biomass per recruit when fished at rate giving MSY divided by that when unfished

SPR_{MSY}
Reference Points of Strategic Fishery Management

Maximum Excess Recruitment

Ratio of MER to that in an unfished population

\[ \frac{MER}{SPR_{MER}} \]
The Stock Assessment Process: Reconstruct the Trajectories of Biomass and Reproduction

![Graph showing the trajectory of biomass and reproduction over time.](image)

**Beginning biomass (m tons)**

**Total biomass**

**Eggs**

YEAR

- 1947
- 1952
- 1957
- 1962
- 1967
- 1972
- 1977
- 1982
- 1987
- 1992
- 1997
- 2002

Beginning biomass peaks around 1977, while the total biomass and eggs show a significant rise and peak around the same period.
The Stock Assessment Process: Forward Project the Fate of the Stock

No fishing
The Stock Assessment Process: Frequency Distribution of Biomass Projected Under Different Scenarios About Fishing
The Stock Assessment Process: Frequency Distribution of Biomass Projected Under Different Scenarios About Fishing

Steepness of the SRR is clearly important in these predictions.
Steepness is Often Fixed In Stock Assessments

Steepness is Often Fixed In Stock Assessments

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Connections Between RPs and Steepness Have Been Noted

Williams (NA J Fish Mang. 2002)

MSY increases with steepness but depends on age at maturity and selectivity.

\( SPR_{MSY} \) determined by steepness but independent of age at maturity and selectivity
Connections Between RPs and Steepness Have Been Noted

Williams (NA J Fish Mang. 2002)

MSY increases with steepness but depends on age at maturity and selectivity.

\( SPR_{MSY} \) determined by steepness but independent of age at maturity and selectivity

Punt et al (Fish. Res. 2008)

\( F_{MSY} \) is an increasing function of steepness but depends on life history parameters and selectivity

\( B_{MSY} / B_0 \) and \( SPR_{MSY} \) nearly perfectly predicted by steepness
Connections Between RPs and Steepness Have Been Noted


\[ SPR_{MER} = \frac{1}{2} \sqrt{\frac{1-h}{h}} \]
Connections Between RPs and Steepness Have Been Noted


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Begs us to answer three questions
Connections Between RPs and Steepness Has Been Noted


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Begs us to answer three questions

1) Why is this so?
Connections Between RPs and Steepness Has Been Noted


\[ SPR_{MER} = \frac{1}{2} \sqrt{\frac{1-h}{h}} \]

Begs us to answer three questions

1) Why is this so?

2) What does it mean for the process of stock assessment?
Connections Between RPs and Steepness Has Been Noted


\[ SPR_{MER} = \frac{1}{2} \sqrt{\frac{1-h}{h}} \]

Begs us to answer three questions

1) Why is this so?

2) What does it mean for the process of stock assessment?

3) What should we do given the answer to 2)?
"Some other observations are worth noting .... For the 14 populations, there was strong negative relationship between $h$ and at [the MSY harvest fraction] under both Beverton-Holt ($r=-0.96$) and Ricker ($r=-0.94$) recruitment (Figs 8a, 8b). This might be expected, as populations with stronger recruitment compensation are expected to be able to be sustained at lower levels of spawning biomass."
The Results of Forrest et al Are Nearly Perfectly Predicted by Steepness

Reported $B_{MSY}/B_0$ by Forrest et al (2010)

Predicted $B_{MSY}/B_0$ from Steepness

Points: Forrest et al

Line: Theory (to be explained)
And Other Results of Forrest et al Are Nearly Perfectly Predicted by Steepness Alone

Points: Forrest et al  
Line: Theory (to be explained)
Some Stock Assessments with BH-SRR and Fixed Steepness

<table>
<thead>
<tr>
<th>Common Name</th>
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<tbody>
<tr>
<td>Cabezon</td>
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Points: SPR estimated from complicated stock assessments.

Line: The theory (to be explained)
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A Theory for The Reproductive Ecology of Steepness: The Production Model

Dynamics of Biomass

\[
\frac{dB}{dt} = \frac{\alpha_p B}{1 + \beta B} - (M + F)B
\]

Steady state in the absence of fishing

\[
B_0 = \frac{1}{\beta} \left( \frac{\alpha_p}{M} - 1 \right)
\]
So that

\[ \frac{\alpha_p}{M} \]

is a dimensionless (Beverton) number of the life history

Steepness is

\[ h = 0.2 \cdot \frac{1 + \beta B_0}{1 + 0.2 \beta B_0} \]
\[ h = 0.2 \cdot \frac{1 + \beta B_0}{1 + 0.2 \beta B_0} \quad \text{and} \quad B_0 = \frac{1}{\beta} \left( \frac{\alpha_p}{M} - 1 \right) \]
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So that
\[ \beta B_0 = \left[ \frac{\alpha_p}{M} - 1 \right] \]
\[ h = 0.2 \cdot \frac{1 + \beta B_0}{1 + 0.2 \beta B_0} \quad \text{and} \quad B_0 = \frac{1}{\beta} \left( \frac{\alpha_p}{M} - 1 \right) \]

So that

\[ \beta B_0 = \left[ \frac{\alpha_p}{M} - 1 \right] \]

And steepness is

\[ h = \frac{\alpha_p}{M} \quad \text{and} \quad 4 + \frac{\alpha_p}{M} \]

Constant for constant Bevorton number
The Age Structured Model

\[ B = \sum_a W_a P_a N(a, t) \]

\[ f(B) \]

\[ N(0, t) \]

\[ N(0, t-1) \rightarrow N(1, t) \]

\[ N(0, t-2) \rightarrow N(1, t-1) \rightarrow N(2, t) \]

\[ N(0, t-3) \rightarrow N(1, t-2) \rightarrow N(2, t-1) \rightarrow N(3, t) \]

\[ \vdots \]

\[ N(0, t-a_{\text{max}}) \rightarrow \ldots \rightarrow N(a_{\text{max}}, t) \rightarrow N(0, t) \]
The Age Structured Model

Population dynamics, beyond eggs/larvae

\[ N(a,t) = N(a-1,t-1)e^{-Z(a-1)} \]
The Age Structured Model

Population dynamics, beyond eggs/larvae

\[ N(a,t) = N(a - 1, t - 1) e^{-Z(a-1)} \]

Spawning biomass

\[ B_s(t) = \sum_{a=1}^{a_{\text{max}}} N(a,t) W_f(a) p_{f,m}(a) \]
The Age Structured Model

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Spawning biomass

\[
B_s(t) = \sum_{a=1}^{a_{\text{max}}} N(a,t)W_f(a)p_{f,m}(a)
\]

Recruits

\[
N(0,t) = \frac{\alpha_s B_s(t)}{1 + \beta B_s(t)}
\]
In the steady state

\[ \bar{N}(a) = S(a) \cdot R_0 \]

\[ R_0 = \frac{\alpha_s B_0}{1 + \beta B_0} \]

\[ B_0 = \sum_{a=1}^{a_{\text{max}}} \bar{N}(a)W_f(a)p_{f,m}(a) \]
In the steady state

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\[ B_0 = \sum_{a=1}^{a_{\text{max}}} \bar{N}(a) W_f(a) p_{f,m}(a) \]

Define

\[ \bar{W}_f = \sum_{a=1}^{a_{\text{max}}} S(a) W_f(a) p_{f,m}(a) \]

Average mass of a spawning fish
\[ R_0 = \frac{\alpha_s R_0 \overline{W}_f}{1 + \beta \cdot R_0 \overline{W}_f} \]
Steepness is

\[ R_0 = \frac{\alpha_s R_0 \bar{W}_f}{1 + \beta \cdot R_0 \bar{W}_f} \]

\[ h = \frac{\alpha_s \cdot 0.2 R_0 \bar{W}_f}{1 + \beta \cdot 0.2 R_0 \bar{W}_f} \]
Steepness is

\[ R_0 = \frac{\alpha_s R_0 \overline{W}_f}{1 + \beta \cdot R_0 \overline{W}_f} \]

But

\[ \beta R_0 \overline{W}_f = \alpha_s \overline{W}_f - 1 \]
Steepness is

\[ R_0 = \frac{\alpha_s R_0 \overline{W}_f}{1 + \beta \cdot R_0 \overline{W}_f} \]

But

\[ \beta R_0 \overline{W}_f = \alpha_s \overline{W}_f - 1 \]

So that

\[ h = \frac{0.2 \alpha_s \overline{W}_f}{1 + 0.2 \left[ \alpha_s \overline{W}_f - 1 \right]} = \frac{\alpha_s \overline{W}_f}{4 + \alpha_s \overline{W}_f} \]
Connecting Age-Structured and Production Models

These are the same if

\[
\frac{\alpha_s \bar{W}_f}{4 + \alpha_s \bar{W}_f} \approx \frac{\alpha_p}{M} + \frac{\alpha_p}{M}
\]
Connecting Age-Structured and Production Models

These are the same if

\[ S(a) = e^{-Ma} \]

\[ \frac{\alpha_s \bar{W}_f}{4 + \alpha_s \bar{W}_f} \approx \frac{\alpha_p}{M} \]

\[ 4 + \frac{\alpha_p}{M} \]

Constant mortality
Connecting Age-Structured and Production Models

These are the same if

\[ S(a) = e^{-Ma} \]

\[ 1 - e^{-Ma_{\text{max}}} \approx 1 \]

Constant mortality

Long-lived
Connecting Age-Structured and Production Models

These are the same if

\[ S(a) = e^{-Ma} \]

\[ 1 - e^{-Ma_{\text{max}}} \approx 1 \]

\[ \frac{1}{1 - e^{-M}} \approx \frac{1}{M} \]

\[ \frac{\alpha_s \bar{W}_f}{4 + \alpha_s \bar{W}_f} \approx \frac{\alpha_p}{M} \]

Constant mortality

Long-lived

M is small
Connecting Age-Structured and Production Models

These are the same if

\[
S(a) = e^{-Ma} \quad \text{Constant mortality}
\]

\[
1 - e^{-Ma_{\text{max}}} \approx 1 \quad \text{Long-lived}
\]

\[
\frac{1}{1 - e^{-M}} \approx \frac{1}{M} \quad M \text{ is small}
\]

\[
W_f(a) \sim \text{constant} \quad \text{Mass at age constant}
\]
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Looking at Some Reference Points for the Production Model

\[
\frac{dB}{dt} = \frac{\alpha_p B}{1 + \beta B} - (M + F)B
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Looking at Some Reference Points for the Production Model

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\bar{B}(F) = \frac{1}{\beta} \left( \frac{\alpha_p}{M + F} - 1 \right)
\]
Looking at Some Reference Points for the Production Model

\[ \frac{dB}{dt} = \frac{\alpha_p B}{1 + \beta B} - (M + F)B \]

\[ \bar{B}(F) = \frac{1}{\beta} \left( \frac{\alpha_p}{M + F} - 1 \right) \]

\[ \bar{Y}(F) = F\bar{B}(F) \]
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What is \( F_{MSY} \)?
Looking at Some Reference Points for the Production Model

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What is \( F_{MSY} \)?

\[
\frac{F_{MSY}}{M} = \sqrt{\frac{\alpha_p}{M}} - 1
\]
Looking at Some Reference Points for the Production Model

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\frac{dB}{dt} = \frac{\alpha_p B}{1 + \beta B} - (M + F)B
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What is $F_{MSY}$?

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F_{MSY} = \sqrt{\frac{\alpha_p}{M}} - 1
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But

\[
h = \frac{\alpha_p}{M} \frac{M}{4 + \frac{\alpha_p}{M}}
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Looking at Some Reference Points for the Production Model

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What is \( F_{MSY} \)?

\[
F_{MSY} = \sqrt{\frac{\alpha_p}{M}} - 1
\]

But

\[
h = \frac{\alpha_p}{M} \quad \quad 4 + \frac{\alpha_p}{M}
\]

so that

\[
\frac{\alpha_p}{M} = \frac{4h}{1 - h}
\]
Thus, elementary calculus, used wisely, shows that:

\[
\frac{F_{MSY}}{M} = \sqrt{\frac{4h}{1-h}} - 1
\]

- Only two of the three parameters natural mortality, steepness, and fishing mortality giving MSY can be treated as freely estimated parameters in an estimation procedure.
Thus, elementary calculus, used wisely, shows that:

\[ \frac{F_{MSY}}{M} = \sqrt{\frac{4h}{1-h}} - 1 \]

- Only two of the three parameters natural mortality, steepness, and fishing mortality giving MSY can be treated as freely estimated parameters in an estimation procedure.

- It is logically inconsistent to try to estimate all three of them in a stock assessment based on the BH-SRR. This inconsistency may show up as poor model fit and be modeled as ‘noise’.
Thus, elementary calculus, used wisely, shows that:

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\frac{F_{MSY}}{M} = \sqrt{\frac{4h}{1-h}} - 1
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- Only two of the three parameters natural mortality, steepness, and fishing mortality giving MSY can be treated as freely estimated parameters in an estimation procedure.

- It is logically inconsistent to try to estimate all three of them in a stock assessment based on the BH-SRR. This inconsistency may show up as poor model fit and be modeled as ‘noise’.

- Furthermore, note that setting steepness equal to 1, makes \( F_{MSY} \) infinite.
More On Reference Points:
Another common management strategy is to choose the fishing mortality rate that produces a steady state biomass that is $x$ per-cent of the unfished biomass ($F_x$).

$$\frac{1}{\beta} \left( \frac{\alpha_p}{M + F_x} - 1 \right) = \frac{x}{\beta} \left( \frac{\alpha_p}{M} - 1 \right)$$
More On Reference Points:

Another common management strategy is to choose the fishing mortality rate that produces a steady state biomass that is \( x \) per-cent of the unfished biomass \((F_x)\).

\[
\frac{1}{\beta} \left( \frac{\alpha_p}{M + F_x} - 1 \right) = \frac{x}{\beta} \left( \frac{\alpha_p}{M} - 1 \right)
\]

Rearranging gives

\[
\frac{\alpha_p}{M} - 1 = x \left( \frac{\alpha_p}{M} - 1 \right)
\]
More On Reference Points:

Another common management strategy is to choose the fishing mortality rate that produces a steady state biomass that is \(x\) per-cent of the unfished biomass \((F_x)\).

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Rearranging gives

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\]

\[
\alpha_p = \frac{4h}{1 - h}
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\frac{\alpha_p}{M} = \frac{4h}{1 - h}
\]

\[
\frac{F_x}{M}
\]

is completely determined by \(x\) and steepness
More Results Another primary management reference point is the biomass that gives maximum net recruitment, \( B_{MNP} \).

\[
\frac{B_{MNP}}{B_0} = \sqrt{\frac{\alpha_p}{M}} - 1 = \sqrt{\frac{4h}{1-h}} - 1
\]

Thus, the single parameter steepness determines both of the major reference management points.
An Example

Suppose in a data-poor stock assessment we assert that $h=0.8$, $M=0.15$. 
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\begin{align*}
B_0 &= 15 / \beta; \\
F_{MSY} &= 3M; \\
B_{MNP} &= 0.2B_0 \\
MSY &= 0.09B_0
\end{align*}
An Example

Suppose in a data-poor stock assessment we assert that $h=0.8$, $M=0.15$. Then we have no choice but to conclude that

$$B_0 = 15 / \beta;$$

$$F_{MSY} = 3M;$$

$$B_{MNP} = 0.2B_0$$

$$MSY = 0.09B_0$$

The only free parameter that can be estimated in this stock assessment is unfished biomass (or alternatively the strength of density dependence of recruitment), and it is clear that its estimated value is strongly conditional on assumed values of $M$ and/or $h$. 
“Okay, that is true for the production model, but not for the age-structured model because of selectivity curves”
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Production Model

\[
h = \frac{\alpha_p}{4 + \frac{\alpha_p}{M}}
\]
“Okay, that is true for the production model, but not for the age-structured model because of selectivity curves”

**Production Model**

\[
F_{MSY} = \frac{\sqrt{4h}}{M} = \sqrt{\frac{4h}{1-h}} - 1
\]
“Okay, that is true for the production model, but not for the age-structured model because of selectivity curves”

Production Model

\[ h = \frac{\alpha_p}{M} \frac{1}{4 + \frac{\alpha_p}{M}} \]

Age Structured Model

\[ h = \frac{\alpha_s \bar{W}_f}{4 + \alpha_s \bar{W}_f} \]

\[ \frac{F_{MSY}}{M} = \sqrt{\frac{4h}{1 - h}} - 1 \]
“Okay, that is true for the production model, but not for the age-structured model because of selectivity curves”

**Production Model**

\[
    h = \frac{\alpha_p}{4 + \frac{\alpha_p}{M}}
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**Age Structured Model**

\[
    h = \frac{\alpha_s \bar{W}_f}{4 + \alpha_s \bar{W}_f}
\]

**Hypothesis:**

\[
    F_{MSY} = \sqrt{\frac{4h}{1 - h}} - 1
\]

\[
    F_{MSY} \bar{W}_f = g(h)
\]
Faster Growth Makes the Age Structured Model More and More Like a Production Model
The Test of the Hypothesis: Scaled Results

Points: Production Model
Lines: Age Structured Model

\[
\frac{F_{MSY}}{W_f} \quad \text{(lines)}
\]

\[
\sqrt{\frac{4h}{1-h}} - 1 \quad \text{(points)}
\]

Constant mortality
Long-lived
\(M\) is small
Mass at age constant
“Your Results Depend on the Fishery Selectivity Curve and Fishing Mortality Giving MSY So Do Not Generalize”

**Spawning Biomass Per Recruit (SBR) at Fishing Mortality F**

\[
SBR(F) = \sum_{a=1}^{20} \bar{N}(a)W(a)p_m(a)\prod_{a'=0}^{a-1}e^{-M(a')-s(a)F}
\]
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**Spawning Potential Ratio (also %MSP)**

\[
SPR_{MSY} = \frac{SBR(F_{MSY})}{SBR(0)}
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**Spawning Potential Ratio (also %MSP)**

\[
SPR_{MSY} = \frac{SBR(F_{MSY})}{SBR(0)}
\]
Spawning Potential Ratio for the Production Model

\[ SPR_{MSY} = \frac{SBR(F_{MSY})}{SBR(0)} = \sqrt{\frac{M}{\alpha}} = \sqrt{\frac{1-h}{4h}} \]

Note

\[ SPR_{MSY} \to 0 \]

as

\[ h \to 1 \]
Another Example: Three Selectivities, Six Ages at Maturity
Another Example: Three Selectivities, Six Ages at Maturity

![Graph showing SPR vs. Steepness, h](image)

**SPR**<sub>MSY</sub> versus **Steepness, h**

- **SPR**<sub>MSY</sub> ranges from approximately 1.0 to 0.0
- **Steepness, h** ranges from 0.2 to 1.0
Outline

• Density Dependence and Population Biology
• What is Steepness?
• Strategic Fishery Management: The Stock Assessment Process
• Connections Between Steepness and Reference Points: Previous Observations
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• Moving Forward
• Conclusions
Fixing Steepness at 1: Biological Interpretation

\[ h = \frac{\alpha_p}{M} \left( 4 + \frac{\alpha_p}{M} \right) \]

\[ h \rightarrow 1 \quad \text{as} \quad \frac{\alpha_p}{M} \rightarrow \infty \]

The stock is either infinitely productive or infinitely long-lived (or both)
Fixing Steepness at 1: Biological Interpretation

\[ h = \frac{\alpha_p}{M\left(4 + \frac{\alpha_p}{M}\right)} \]

As \( \frac{\alpha_p}{M} \to \infty \)

\( h \to 1 \)

The stock is either infinitely productive or infinitely long-lived (or both)

There is no evidence yet of Darwinian demons in fish
Fixing Steepness at 1: Biological Interpretation (continued)

\[
\frac{F_{MSY}}{M} = \sqrt{\frac{4h}{1-h}} - 1
\]

As \( h \to 1 \)

\[
\frac{F_{MSY}}{M} \to \infty
\]

Since the stock is infinitely productive, we can fish it infinitely hard!!!
Fixing Steepness at 1: Probabilistic Interpretation

\[ h = 1 \] means

\[ \Pr\{R(0.2B_0) = R_0\} = 1 \]

With certainty, recruitment at 20% of unfished biomass is unfished recruitment. Is this what we mean?
Fixing Steepness at 1: Probabilistic Interpretation

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What if recruitment at 20% of unfished biomass can take any value between 20% of unfished recruitment and unfished recruitment?
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Steepness ranges between 0.2 and 1.0 (tied down at both ends by biology)
Thus, steepness should have a probability distribution.

Michielsens and McAllister. 2004. CJKAS 61:1032-1047

Harley et al. 2009. Bigeye Tuna Assessment
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Three Options for Moving Forward

Do Not Fix Steepness and Mortality Rate
Three Options for Moving Forward

Do Not Fix Steepness and Mortality Rate

Replace the BH-SRR by a SRR that Avoids the Problem
Three Options for Moving Forward

Do Not Fix Steepness and Mortality Rate

Replace the BH-SRR by a SRR that Avoids the Problem

Be fully honest about the limitations of the data and the stock assessment
Do Not Fix Steepness and Mortality Rate

When will data be informative?

Use Simulation Methods to determine what kinds of data are necessary so that steepness and natural mortality can be estimated in the stock assessment.
Do Not Fix Steepness and Mortality Rate

When will data be informative?

Use Simulation Methods to determine what kinds of data are necessary so that steepness and natural mortality can be estimated in the stock assessment

Already started:


Replace the BH-SRR by a SRR that Avoids the Problem

An example: Maynard Smith/Shepherd model

\[
\frac{dB}{dt} = \frac{\alpha_p B}{1 + \beta B^n} - (M + F)B
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Replace the BH-SRR by a SRR that Avoids the Problem

An example: Maynard Smith/Shepherd model

\[ \frac{dB}{dt} = \frac{\alpha_p B}{1 + \beta B^n} \left( M + F \right) B \]
\[
\frac{dB}{dt} = \frac{\alpha_p B}{1 + \beta B^n} - (M + F)B
\]

- \(n > 1\), Cushing-like
- \(n = 1\), Beverton-Holt
- \(n < 1\), Ricker-like

**Recruitment, \(S(B)\)**

**Spawning Stock Biomass, \(B\)**
For Maynard Smith Shepherd SRR, steepness is

\[ h = \frac{0.2 \cdot \frac{\alpha}{M}}{1 + 0.2^{1/n} \left( \frac{\alpha}{M} - 1 \right)} \]
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so that

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\frac{\alpha}{M} = \frac{h(1 - 0.2^{1/n})}{0.2 - 0.2^{1/n} h}
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and most importantly

\[ \frac{F_{MSY}}{M} = \frac{\frac{\alpha}{M} (1 - n) + \sqrt{\left( \frac{\alpha}{M} \right)^2 (1 - n)^2 + 4 \frac{\alpha n}{M}}}{2} - 1 \]
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\[ F_{MSY} = \frac{\frac{\alpha}{M} (1 - n) + \sqrt{\left( \frac{\alpha}{M} \right)^2 (1 - n)^2 + 4 \frac{\alpha n}{M}}}{2} - 1 \]

There is still an undetermined parameter for the analysis -- the data can tell us something! - or we can integrate over the potential range of \( n \)
Application to Cowcod (Dick, WGC 2012)

\[ \frac{B_{MSY}}{B_0} \]

\[ \frac{F_{MSY}}{M} \]
Application to Cowcod (Dick, WGC 2012)

3 parameter SRR

BH SRR ($h$ fixed at 0.6)
We Can Do a Meta-analysis on the Parameter, And It Might Look Like This

Distribution of \( n \)
Or The Meta-analysis Could Yield This
Recruitment, $S(B)$ vs Spawning Stock Biomass, $B$
Be fully honest about the limitations of the data and the stock assessment

• Management policy with biomass targets or rebuilding plans on a fixed time table with specified probability may often overstep what can realistically be expected from a defensible assessment of an individual stock.
Be fully honest about the limitations of the data and the stock assessment

• Management policy with biomass targets or rebuilding plans on a fixed time table with specified probability may often overstep what can realistically be expected from a defensible assessment of an individual stock.

• The community of stock assessment scientists needs to agree on workable protocols for several classes of life history parameters, ecosystem types, fishery histories that are reasonably robust in achieving management objectives in the face of scientific uncertainty. Developing good proxies and protocols is a scientific matter. It should be based on meta analyses and management strategy evaluation.
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Conclusions, Part 1: A scientific theory should be as simple as possible, but no simpler.

“The intuitive mind is a sacred gift and the rational mind is a faithful servant. We have created a society that honors the servant and has forgotten the gift.”

~Albert Einstein~
**Conclusions, Part 1:** A scientific theory should be as simple as possible, but no simpler.

_This is a case of a too simple theory_
Conclusions, Part 2

- As soon as we are able to develop a demographic model for the survival and reproduction of a cohort, we are able to obtain a point estimate for steepness (steepness cannot simply be fixed)
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• Allowing stochasticity in survival allows us to derive probability distributions, not just point estimates (see Mangel et al. 2010. Fish and Fisheries 11:89-104; NMFS Toolbox Code in Development)
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• The early life history is key to understanding steepness

• Environmental forcing can be built into the early life history through fluctuations in mortality rate and into productivity through fluctuations in egg production.
Conclusions, Part 3: Setting Steepness Equal to 1 is

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  Recruitment independent of stock size does not mean $h=1$ (with probability 1 the recruitment at 20% of spawning stock size is the same as $R_0$). Rather it means *any* percentage of recruitment is possible: $h$ has a uniform distribution (tied down at small values and near 1).
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Figure 9. Spawner-recruitment relationship based on Base case estimates.
The first responsibility of scientists is to the integrity of science and it is critical to be explicit about what is known and not known. And there is not a moment to be lost.
A Few Citations


